Counterintuitive approximations

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The Nash-Kuiper embedding theorem is a prototypical example of a counterintuitive approximation result: any short embedding of a Riemannian manifold into Euclidean space can be approximated by \textbf{isometric} ones. As a consequence, any surface can be isometrically $C^1$-embedded into an arbitrarily small ball in $\mathbb{R}^3$. For $C^2$-embeddings this is impossible due to curvature restrictions.

I will present a general result which allows for approximations by functions satisfying strongly overdetermined equations on open dense subsets. This will be illustrated by three examples: Lipschitz functions with surprising derivative, surfaces in 3-space with unexpected curvature properties, and a similar statement for abstract Riemannian metrics on manifolds.

Our method is based on ”cut-off homotopy”, a concept introduced by Gromov in 1986. This is joint work with Bernhard Hanke.

References