Continuous Wavelet Transform of (subrepresentations of) the left-regular representation

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Let $(\pi, \mathcal{H})$ be a unitary representation of a (second countable) locally compact group G. For a vector $\psi \in \mathcal{H}$, the (possibly unbounded) operator $V_\psi \colon \mathcal{H} \to L^2(G)$ given by $V_\psi f(x) = \langle \pi(x)\psi, f\rangle$ is called Continuous Wavelet Transform (CWT) if V_ψ is an isometry. [1]

The CWT has applications for instance in microlocal analysis (when studying singularities) as well as in applied mathematics (data analysis, data compression), where it is used as a starting point for the Discrete Wavelet Transform.

A necessary condition for admitting a CWT is that π is a subrepresentation of the left-regular representation of G. For non-unimodular groups this condition is already sufficient whereas unimodular groups do have subrepresentations admitting no CWT. In the talk I generalize the definition of a CWT to an isometry $\mathcal{H} \to L^2(G \times \mathbb{N})$ and show that using this definition every subrepresentation of the left-regular representation admits a generalized CWT. The idea for this result is based on [2].

References

[1] H. Führ, *Abstract Harmonic Analysis of Continuous Wavelet Transforms*, Lecture Notes in Mathematics **1863**, Springer, Berlin, 2005.

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