On the factors of CNS polynomials

Horst Brunotte[1,*]

[1] *none*

*Email: `brunoth@web.de`

Extending several known number systems A. Pethő and J. Thuswaldner [1] recently introduced and studied the notion of a generalized number system (GNS) over an order $\mathcal{O}$ in an algebraic number field. A GNS is a pair $(p, \mathcal{D})$ where p is a monic univariate polynomial with coefficients in $\mathcal{O}$ and $p(0) \neq 0$ and $\mathcal{D} \subset \mathcal{O}$ is a complete residue system modulo $p(0)$. The GNS $(p, \mathcal{D})$ enjoys the finiteness property if every polynomial in $\mathcal{O}[X]$ is congruent modulo p to some polynomial in $\mathcal{D}[X]$. An example of a GNS is the canonical number system (CNS) which has been introduced by A. Pethő. Recall that an integer polynomial $p \in \mathbb{Z}[X]$ is a CNS polynomial if $(p, \{0, \ldots, |p(0)| - 1\})$ is a GNS with finiteness property.

Many years ago, A. Pethő asked whether each monic integer polynomial all of whose roots lie outside the closed unit disk and are non-positive is a factor of a CNS polynomial. We give an affirmative answer to this question by a constructive proof based on a classical result by S. Akiyama – H. Rao and K. Scheicher – J. M. Thuswaldner.

References

[1] A. Pethő and J. Thuswaldner, Number systems over orders, *Monatsh. Math.* **187** (2018), pp. 681–704.