The non-linear Brascamp-Lieb inequality

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We prove a nonlinear variant of the general Brascamp-Lieb inequality. Instances of this inequality are quite prevalent in analysis, and we illustrate this with substantial applications in the theory of oscillatory integrals, abstract harmonic analysis and partial differential equations. Our proof consists of running an efficient, or "tight", induction on scales argument, which uses the existence of Gaussian near-extremisers to the underlying linear Brascamp-Lieb inequality (Lieb's theorem) in a fundamental way. A key ingredient is an effective version of Lieb's theorem, which we establish via a careful analysis of near-minimisers of weighted sums of exponential functions. This is joint work with Jon Bennett, Neal Bez, Michael Cowling and Taryn Flock.