Unique Continuation for the Zakharov Kuznetsov equation

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In this talk we analyze uniqueness properties of solutions to the Zakharov-Kuznetsov (ZK) equation

$$\partial_t u + \partial_x^3 u + \partial_x \partial_y^2 u + u \partial_x u = 0, \qquad (x,y) \in \mathbb{R}^2, \quad t \in [0,1].$$

Mainly motivated by the very well known PDE's counterpart of the Hardy uncertainty principle, we provide a two times unique continuation result. More precisely, we prove that given u_1, u_2 two solutions to ZK, as soon as the difference $u_1 - u_2$ decays (spatially) fast enough at two different instants of time, then $u_1 = u_2$. As expected, it turns out that the decay rate needed to get uniqueness reflects the asymptotic behavior of the fundamental solution of the associated linear problem. Encouraged by this fact we also prove optimality of the result.

Some recent results concerning the $(3+1)$-dimensional ZK equation will be also presented.

The seminar is based on a recent paper [1] in collaboration with L. Fanelli and F. Linares.

References

[1] L. Cossetti, L. Fanelli and F. Linares, Uniqueness results for Zakharov-Kuznetsov equation, *Comm. Partial Differential Equations*, DOI:10.1080/03605302.2019.1581803