# The $p$-adic zeta function and a $p$-adic Euler constant

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We study the $p$-adic analogue $\gamma_p$ of the Euler-Mascheroni constant $\gamma$, also known as Euler constant. The $p$-adic Euler constant can be defined using the $p$-adic analogue of the gamma function. The constant $\gamma_p$ can also be expressed in terms of the Kubota-Leopoldt $p$-adic $L$-function: $\gamma_p$ is the constant term in the Laurent series expansion of the $\chi = 1$ branch of the $p$-adic zeta function about $s = 1$ (see [1]).

The $p$-adic zeta function can be constructed using $p$-adic distributions or measures and there are different series expansions (see [2], [3]). We derive several formulas for $\gamma_p$ (compare [3], [4]) and present computations with SageMath.

## References

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