On the eigenvalues of the Robin Laplacian with a complex parameter

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We study the spectrum of the Robin Laplacian $-\Delta_\Omega^\alpha$ with a complex parameter α on a bounded Lipschitz domain $\Omega \subset \mathbb{R}^d$. We establish a number of properties, in particular regarding the analytic dependence of eigenvalues and eigenspaces on $\alpha \in \mathbb{C}$ as well as basis properties of the eigenfunctions. Using estimates on the numerical range of the associated operator we give bounds and asymptotics for the eigenvalues as functions of α, which lead to new eigenvalue bounds even in the self-adjoint case $\alpha \in \mathbb{R}$. For the asymptotics of the eigenvalues as $\alpha \to \infty$, in place of the variational min-max characterisation of the eigenvalues and Dirichlet-Neumann bracketing techniques commonly used in the self-adjoint case, we exploit the duality between the eigenvalues of the Robin Laplacian and the eigenvalues of the Dirichlet-to-Neumann map to show that every Robin eigenvalue either diverges to ∞ in $\mathbb{C}$ or converges to a point in the Dirichlet spectrum.