Periodicities for Taylor coefficients of half-integral weight modular forms

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Congruences of Fourier coefficients of modular forms have long been an object of central study. By comparison, the arithmetic of other expansions of modular forms, in particular Taylor expansions around points in the upper-half plane, has been much less studied. Recently, Romik made a conjecture about the periodicity of coefficients around $\tau_0 = i$ of the classical Jacobi theta function θ_3 . Here, we generalize the phenomenon observed by Romik to a broader class of modular forms of half-integral weight and, in particular, prove the conjecture.