

# CSPs of $\omega$ -categorical algebras

**Thomas Quinn-Gregson<sup>1,\*</sup>, Manuel Bodirsky<sup>1</sup>**

<sup>1</sup>*Institute of Algebra, TU Dresden, Dresden, Germany*

\*Email: [thomas.quinn-gregson@tu-dresden.de](mailto:thomas.quinn-gregson@tu-dresden.de)

Given an algebra  $A$ , we are concerned with the following computational complexity problem:

1. Instance: a finite list  $E$  of equations and disequalities over  $A$  with variables from a finite set  $V$ .
2. Question: is there an assignment  $\phi : V \rightarrow A$  such that  $E$  holds in  $A$ ?

For example, if  $A$  is a semigroup then an instance could be  $\{xy = z, zy = t, y \neq t\}$ . We are mainly concerned with  $\omega$ -categorical algebras. The constraint satisfaction problem (CSP) for  $\omega$ -categorical structures is well-studied, and we shall show that the algebras we consider give rise to CSPs with ‘well behaved’ templates. A number of algebras have already been considered; the atomless Boolean algebra is  $\text{NP}$ -hard (Bodirsky, Hils, Krimkevitch) while the in

nite dimensional vector space over  $\mathbb{F}_q$  is tractable (Bodirsky, Chen, Kára, von Oertzen). We extend the latter work by classifying the tractable  $\omega$ -categorical abelian groups. A necessary condition for the tractability of a CSP over an  $\omega$ -categorical structure is the existence of a pseudo-Siggers polymorphism. We show that for semilattices and lattices, the existence of such a polymorphism is a useful restriction, and allows us to consider only semilattices and lattices which are bi-embeddable with their direct power. As a consequence, we are able to classify tractable  $\omega$ -categorical semilattices.