The spectral problem associated with the time-periodic NLS

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According to its Lax pair formulation, the nonlinear Schrödinger equation (NLS) can be expressed as the compatibility condition of two linear ordinary differential equations with an analytic dependence on a complex parameter. The first of these equations—often referred to as the *x-part* of the Lax pair—can be rewritten as an eigenvalue problem for a Zakharov-Shabat operator. The spectral analysis of this operator is crucial for the solution of the initial value problem for NLS via inverse scattering techniques. For space-periodic solutions, this leads to the existence of a Birkhoff normal form, which beautifully exhibits the structure of NLS as an infinite-dimensional completely integrable system. In this talk we present several aspects of the recent work [1], in which we take the crucial steps towards developing an analogous picture for time-periodic solutions by performing a spectral analysis for the *t-part* of the Lax pair with periodic potentials. In particular, we discuss the asymptotics of the fundamental matrix solution of the underlying generalized eigenvalue problem for arbitrary potentials, which implies estimates for large periodic eigenvalues. Furthermore we present qualitative aspects of the periodic spectrum for small periodic potentials of so-called real or imaginary type being particularly relevant for NLS. Finally we illustrate these results with the help of a single exponential potential whose corresponding fundamental solution exhibits an explicit formula.

References

[1] J. Lenells and R. Quirchmayr, On the spectral problem associated with the time-periodic nonlinear Schrödinger equation, *Math. Ann.* (2019), DOI: 10.1007/s00208-019-01856-x, 72pp.