The Hilbert transform along curves, anisotropically homogeneous multipliers and the polynomial Carleson theorem

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In 1966, Fabes and Riviere investigated L^p bounds for Calderón-Zygmund operators with suitable anisotropic homogeinity, in relation to regularity questions for parabolic equations. Inspired by that, E. Stein asked when the Hilbert transform along a smooth curve is L^p-bounded, in terms of fundamental curvature properties of the curve.

Many subsequent works address this question, but we shall mainly focus on a celebrated result with many different proofs: the (two-dimensional) parabolic Hilbert transform given by

$$Tf(x,y) = p.v. \int_{\mathbb{R}} f(x-t, y-t^2) \frac{dt}{t}$$

is bounded in every L^p space for $1 < p < +\infty$.

Passing to the multiplier side, its symbol satisfies a quadratic anisotropic 0-homogeneity relationship. In connection to the classical Carleson theorem, one may ask whether Carleson-like operators associated to this transformation share similar bounds, as the usual Carleson symbol belongs to a linear 0-homogeneous class.

This has been an open problem in time-frequency analysis for the past decade, with particularly intense activity in the past 3 years. In this talk, we shall discuss recent developments on the topic, as well as draw connections between this problem and the celebrated polynomial Carleson theorem of Lie and Zorin-Kranich in order to obtain concrete consequences. As a by-product, we obtain a new proof of a special case of the polynomial Carleson theorem.