The Plato and Gödel hierarchies

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We introduce the *Gödel hierarchy* to be found in e.g. [2], a linear order of logical systems claimed to capture all natural and foundationally important systems. *Second-order arithmetic* constitutes the medium range of the Gödel hiearchy, including the main 'Big Five' systems of *Reverse Mathematics* and associated equivalent theorems (see [1]). We formulate a parallel hierarchy based on *higher-order arithmetic*, dubbed the *Plato Hierarchy*, that yields the medium range of the Gödel hierarchy under the canonical 'ECF' embedding of second- into higher-order arithmetic. In this way, ECF maps convergence results pertaining to *nets* over Baire space to similar results about *sequences*, while Riemann integration theory is obtained from gauge integration theory via ECF. Equivalences involving these higher-order theorems yield the familiar equivalences from Reverse Mathematics.

References

[1] Stephen Simpson, *Subsystems of second order arithmetic*, 2009, Cambridge University press, pp. xvi+444

[2] Stephen Simpson, Reverse Mathematics and the Gödel hierarchy, *Kurt Gödel. Essays for his centennial*, 2010, Cambridge University Press, pp. 109–127,