General Equations for the Classical Groups in Differential Galois Theory

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In classical Galois theory there is the well-known construction of the general equation with Galois group the symmetric group $S_n$. One starts with $n$ indeterminates $T = (T_1, \ldots, T_n)$ and considers the rational function field $\mathbb{Q}(T)$. The group $S_n$ acts on $\mathbb{Q}(T)$ by permuting the indeterminates $T_1, \ldots, T_n$. One can show that $\mathbb{Q}(T)$ is a Galois extension of the fixed field $\mathbb{Q}(T)^{S_n}$ for a polynomial equation of degree $n$ whose coefficients are the elementary symmetric polynomials and are algebraically independent over $\mathbb{Q}$. A generalisation of this idea leads to the so-called Noether problem.

In this talk we perform a similar construction in differential Galois theory for the classical groups of Lie type. Let $G$ be one of them and denote by $\ell$ its Lie rank. Using the geometrical structure of $G$ we build a general differential extension field $E$ of differential transcendence degree $\ell$ and define a group action of $G$ on $E$. We show that the field of invariants $E^G$ is a purely differential transcendental extension of the field of constants of degree $\ell$ and that $E$ over $E^G$ is a Picard-Vessiot extension with group $G$ for a nice linear differential equation. Finally we discuss for which Picard-Vessiot extensions our construction is generic.