General Equations for the Classical Groups in Differential Galois Theory

Matthias Seiß[1,*]

[1] *Universität Kassel, Institut für Mathematik*

[*]Email: mseiss@mathematik.uni-kassel.de

In classical Galois theory there is the well-known construction of the general equation with Galois group the symmetric group S_n. One starts with n indeterminates $T = (T_1, \dots, T_n)$ and considers the rational function field $\mathbb{Q}(T)$. The group S_n acts on $\mathbb{Q}(T)$ by permuting the indeterminates $T_1, \dots, T_n$. One can show that $\mathbb{Q}(T)$ is a Galois extension of the fixed field $\mathbb{Q}(T)^{S_n}$ for a polynomial equation of degree n whose coefficients are the elementary symmetric polynomials and are algebraically independent over $\mathbb{Q}$. A generalisation of this idea leads to the so-called Noether problem.

In this talk we perform a similar construction in differential Galois theory for the classical groups of Lie type. Let G be one of them and denote by ℓ its Lie rank. Using the geometrical structure of G we build a *general* differential extension field E of differential transcendence degree ℓ and define a group action of G on E. We show that the field of invariants E^G is a purely differential transcendental extension of the field of constants of degree ℓ and that E over E^G is a Picard-Vessiot extension with group G for a *nice* linear differential equation. Finally we discuss for which Picard-Vessiot extensions our construction is generic.