Ax-Kochen-Ershov principle and classification of theories of henselian valued fields.

Pierre Touchard[1,*]

[1]*Department for Mathematical Logic and Foundational Research, Münster Universität*
[*]Email: `touchard@uni-muenster.de`

In the 60's, Ax and Kochen gave an approximate solution of the (disproved) Artin conjecture: they showed that for every integer d and for every prime p large enough, all homogeneous polynomials with coefficients in $\mathbb{Q}_p$ of degree d and with more than d^2 variables have a non trivial zero. This is the consequence of their results - independently proved by Ershov, and known as the Ax-Kochen-Ershov (AKE) principle: henselian valued fields of residue characteristic 0 are elementary equivalent if and only if their residue fields are elementary equivalent and their value groups are elementary equivalent. Since, many results echo this principle. In particular, model theorists try to reduce the problem of classification of theories of henselian valued fields to that of their residue fields and that of their value groups.

We will give a brief overview of the problem of classification of first order theories and see how the ideas behind the AKE principle have evolved to answer this problem.