Long-time behaviour of non-local in time Fokker-Planck equations via the entropy method

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We consider a rather general class of non-local in time Fokker-Planck equations and show by means of the entropy method that as $t \to \infty$ the solution converges in L_1 to the unique steady state. Important special cases are the time-fractional and ultraslow diffusion case. We also provide estimates for the rate of decay. In contrast to the classical (local) case, where the usual time derivative appears in the Fokker-Planck equation, the obtained decay rate depends on the entropy, which is related to the integrability of the initial datum. It seems that higher integrability of the initial datum leads to better decay rates and that the *optimal decay rate* is reached, as we show, when the initial datum belongs to a certain weighted L_2 space. Our estimates can be adapted to the discrete-time case thereby improving known decay rates from the literature.

References

[1] J. Kemppainen, R. Zacher, Long-time behavior of non-local in time Fokker-Planck equations via the entropy method. *Math. Models Methods Appl. Sci.* **29** (2019), pp. 209–235.