Some problems with formalisations of mathematical proofs by means of sentential-logical derivations and their solutions

Alexander Zimmermann[1,*]

[1] *University College of Teacher Education Burgenland, Eisenstadt, Austria*

[*] Email: alexander.zimmermann@ph-burgenland.at

What, from a logical point of view, applies to grounding in general, applies to mathematical grounding and thus to mathematical proofs in particular. It is indispensable for an adequate logical formalisation of mathematical proofs to distinguish between the requirements on mathematical proofs and the requirements on mere derivations in a formal system. For, in mere derivations, transitions may occur that would never be accepted in mathematical proofs. Although relevance-logical systems filter out several unacceptable or problematic transitions, their axioms and rules are far from sufficient to guarantee adequate formalisations of mathematical proofs within such systems.

I will propose a definition that allows to formalise mathematical proofs as relations between sets of formulae and formulae in such a way that a set T of sentential-logical formulae grounds mathematically a sentential-logical formula A from a syntactical point of view, if and only if A is a syntactical sentential-logical consequence of T and specific additional syntactical requirements regarding T and A are fulfilled. These additional requirements are strictly developed within the syntactics of sentential-logical languages. The three most important of these requirements are new: to be atomically minimal, to be minimal in degree, and not to be conjunction-like. This approach has, among other things, the advantage to be independent of a special sentential-logical calculus. The next steps should be to do the same for predicate-logical formulae. By means of these definitions, it should be possible to formalise mathematical proofs adequately within formal-logical systems.