

# Berge hypergraphs - saturation and Ramsey properties

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For a graph  $G$ , a hypergraph  $H$  is called Berge- $G$  if for some isomorphic copy  $G'$  of  $G$  and a bijection  $f$  from the edge set  $E(G')$  to the hyperedge set  $E(H)$ , for each  $e \in E(G')$ , we have  $e \subseteq f(e)$ . A hypergraph  $F$  is Berge- $G$  saturated if it does not contain a Berge- $G$  subhypergraph but adding any new hyperedge of size at least two to  $F$  results in such a subhypergraph. Any Berge- $G$  saturated hypergraph has at least  $|E(G)| - 1$  hyperedges. We show that there are saturated Berge- $G$  hypergraphs of size exactly  $|E(G)| - 1$  for all but a few graphs  $G$ . Further, we show that Berge- $G$  hypergraphs possess nice Ramsey properties - their monochromatic copies are unavoidable in edge-colored complete hypergraphs. In particular, we determine the multicolor Ramsey number for Berge triangle asymptotically.