Non-selfsimilar collapse of surface quasi-geostrophic point-vortices

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Point vortex models are presented for the surface quasi-geostrophic (SQG) equations, which are characterized by a fractional Laplacian relation between the active scalar and the streamfunction and for which the existence of finite-time singularities is still a matter of debate. Point vortex trajectories are expressed using Nambu dynamics. The formulation is based on a noncanonical bracket and allows for a geometrical interpretation of trajectories as intersections of level sets of the Hamiltonian and Casimir. Within this setting, we focus on the collapse of solutions for the three point vortex model. In particular, we show that for SQG the collapse can be either self-similar or nonself-similar. Self-similarity occurs only when the Hamiltonian is zero, while non-selfsimilarity appears for non-zero values of the same. For both cases, collapse is allowed for any choice of circulations within a permitted interval. These results differ strikingly from the classical point vortex model. Results may also shed a light on the formation of singularities in the SQG partial differential equations, where the singularity is thought to be reached only in a self-similar way.

References

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