Dynamics of the Selkov oscillator

Pia Brechmann[1,*], **Alan D. Rendall**[1]

[1]*Institut für Mathematik, Johannes Gutenberg-Universität, Mainz, Germany*

[*]Email: `pibrechm@uni-mainz.de`

The Selkov oscillator is a simple mathematical model, a system of two ordinary differential equations, describing the metabolic pathway of glycolysis. Glycolysis is a central part of the energy metabolism that almost all living organisms have in common. Selkov's model was one of the first to mathematically describe the autonomous oscillations observed in experiments under constant substrate supply. To complete the analysis of the system and be able to identify all possible phase portraits, we studied the long term behavior of the solutions via a Poincaré compactification. The model obeys the expectations on a biological oscillator insofar as if there exists a periodic solution it is stable. In addition if the unique steady state is stable all bounded solutions eventually converge to it. At the same time it is the case that irrespective of the choice of parameters there are always solutions tending to a point at infinity. Furthermore it turns out that if the phase portrait does not correspond to one of those above then all solutions except the steady state either tend to a point at infinity or oscillate in a way that every variable takes on arbitrarily large and small values.