

Minimising the Helfrich Energy

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The Helfrich Energy is defined as $\int_{\Sigma} |H - H_0|^2 dA$ for some 2-dimensional smooth oriented manifold $\Sigma \subset \mathbb{R}^3$ and $H_0 \in \mathbb{R}$ some parameter. Here H is the mean curvature of Σ , i.e. the sum of the principal curvatures (see [3] for the original definition and some applications in biology). We will minimise the Helfrich energy under some constraints, for example prescribing Dirichlet boundary data. Compactness is easily achieved in the class of oriented varifolds.

Unfortunately by a counterexample of Große-Brauckmann (see [2]) the Helfrich Energy is in general not lower-semicontinuous with respect to varifold convergence. We are able to circumvent this problem by showing a lower-semicontinuity estimate for the minimising sequence itself (see [1]). This allows us to show some existence results of minimisers in the above mentioned classes.

References

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- [3] W. Helfrich, Elastic properties of lipid bilayers: Theory and possible experiments. *Z. Naturforsch. C*, 28:693–703, 1973.