Lorentzian warped products with one dimensional base and length space fibers

Stephanie B. Alexander\textsuperscript{1}, Melanie Graf\textsuperscript{2,*}, Michael Kunzinger\textsuperscript{3}, Clemens S"amann\textsuperscript{3}

\textsuperscript{1}Department of Mathematics, University of Illinois at Urbana-Champaign, USA
\textsuperscript{2}Department of Mathematics, University of T"ubingen, Germany
\textsuperscript{3}Faculty of Mathematics, University of Vienna, Austria
\textsuperscript{*Email: graf@math.uni-tuebingen.de}

Smooth Lorentzian warped products of the form $I \times f (M, g)$, where $(M, g)$ is a Riemannian manifold and $f$ is a positive smooth function on an interval $I$, are important examples of spacetimes: They contain well-known physical models (such as the FLRW spacetimes) and admit a very simple description of causal curves and geodesics.

We will examine what happens if one replaces the Riemannian manifold $(M, g)$ with a locally compact length space $(X, d)$. As long as $f$ is continuous and positive there still exists a natural notion of causal curves and their length and hence also of the causality relations on the product $I \times f X$. This turns $I \times f X$ into a Lorentzian length space (as defined in \cite{1}). Analogous to the smooth case the causal structure of such warped products is very simple and one has an explicit description of $\partial J^+(p)$. Inspired by the well-developed Riemannian theory of warped products of length spaces, we also obtain some results concerning timelike curvature bounds for Lorentzian warped products of this type.

References