Lorentzian warped products with one dimensional base and length space fibers

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Smooth Lorentzian warped products of the form $I \times_f (M, g)$, where (M, g) is a Riemannian manifold and f is a positive smooth function on an interval I, are important examples of spacetimes: They contain well-known physical models (such as the FLRW spacetimes) and admit a very simple description of causal curves and geodesics.

We will examine what happens if one replaces the Riemannian manifold (M, g) with a locally compact length space (X, d). As long as f is continuous and positive there still exists a natural notion of causal curves and their length and hence also of the causality relations on the product $I \times_f X$. This turns $I \times_f X$ into a Lorentzian length space (as defined in [1]). Analogous to the smooth case the causal structure of such warped products is very simple and one has an explicit description of $\partial J^+(p)$. Inspired by the well-developed Riemannian theory of warped products of length spaces, we also obtain some results concerning timelike curvature bounds for Lorentzian warped products of this type.

References

 M. Kunzinger and C. Sämann, Lorentzian length spaces, Ann. Glob. Anal. Geom. 54 (2018), pp. 399–447.