Regularity of Lie Groups

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In a survey article from 1984, Milnor formulated the regularity concept for infinite-dimensional Lie groups (a la Bastiani) to enable proofs of fundamental Lie theoretical facts in infinite dimensions. Roughly speaking, regularity is concerned with definedness and smoothness/continuity of the product integral – a concept that naturally generalizes the notion of the Riemann integral for curves in locally convex vector spaces to (infinite-)dimensional Lie groups (Lie algebra valued curves are thus integrated to Lie group elements). For instance, the exponential map of a Lie group is the restriction of the product integral to constant curves; and, given a principal fibre bundle, holonomies are product integrals of Lie algebra valued curves that are pairings of smooth connections with derivatives of curves in the base manifold of the bundle.

Although individual arguments show that all known example classes of infinite dimensional Lie groups admit regularity, only recently general regularity criteria had been found. We present these results, including a complete solution to the differentiability (smoothness) issue that forms a substantial part of the regularity problem. We furthermore discuss integrability conditions for Lie algebra valued curves (domain of the product integral), and show that C^0-continuity of the product integral is equivalent to a generalized triangle inequality involving the Lie group multiplication. As a real-life application, we will discuss the strong Trotter property in the given context.