Characterising k-connected sets in infinite graphs

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While the connectivity of a graph is a global invariant, even graphs of low connectivity might contain objects that are highly connected in a certain way. One such type of highly connected objects are k-connected sets:

Given a graph G we define a *k-connected set*, where $k > 0$ is an integer, to be a vertex set $X \subseteq V(G)$ such that any two of its subsets of the same size $\ell \leq k$ can be connected by ℓ disjoint paths in G.

For finite graphs the existence of k-connected sets has already been characterised in terms of unavoidable minors and via certain tree-decompositions, but for infinite graphs similar characterisations were not completely known. In this talk I will discuss our results [1] and the involved proof ideas. This includes a characterisation for the existence of k-connected sets of arbitrary but fixed infinite cardinality via the existence of certain minors and topological minors. In particular, I will address the difficulties occurring when dealing with singular instead of regular infinite cardinals. Moreover, we proved a duality theorem for the existence of such k-connected sets: if a graph contains no such k-connected set, then it has a tree structure which, whenever it exists, precludes the existence of such a k-connected set.

References

[1] J.P. Gollin and K. Heuer, Characterising k-connected sets in infinite graphs, preprint (2018), `https://arxiv.org/abs/1811.06411`.