

The infinitesimal topos as a classifying topos

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Some of the toposes from algebraic geometry have long been known to classify certain geometric theories: The (big) Zariski topos classifies local rings, the étale topos classifies separably closed local rings. There are other important topologies on the Zariski site yielding further subtoposes of the big Zariski topos corresponding to further adjectives before the words “local rings”.

The case of the crystalline topos and infinitesimal topos is different in that they are not subtoposes of the Zariski topos. For the classified theory this means that there must be additional structure instead of only additional axioms. Indeed, we present the result that the big infinitesimal topos classifies nilpotent thickenings of local rings.

As an important tool we use geometric theories of presheaf type (meaning that the classifying topos can be given as a presheaf topos) as these allow for a convenient description of their classifying topos in terms of their Set-based models. To this end, we utilize different techniques to exhibit intermediate theories as theories of presheaf type.