The one-dimensional KPP equation driven by space-time white noise

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The one-dimensional KPP equation driven by space-time white noise,

$$\partial_t u = \partial_{xx} u + \theta u - u^2 + u^{\frac{1}{2}} dW, \qquad t > 0, x \in \mathbb{R}, \theta > 0, \qquad u(0,x) = u_0(x) \geq 0$$

is a stochastic partial differential equation (SPDE) that exhibits a phase transition for initial non-negative finite-mass conditions. Solutions to this SPDE can be seen as the high density limit of particle systems which undergo branching random walks with an extra death-term due to competition or overcrowding. They arise for instance as (weak) limits of approximate densities of occupied sites in rescaled one-dimensional long range contact processes.

If θ is below a critical value θ_c, solutions die out to 0 in finite time, almost surely. Above this critical value, the probability of (global) survival is strictly positive. Let $\theta > \theta_c$, then there exist stochastic wavelike solutions which travel with positive linear speed. For initial conditions that are "uniformly distributed in space", the corresponding solutions are all in the domain of attraction of a unique non-zero stationary distribution.

In my talk, I will introduce the model in question and give an overview of existing results, open questions and techniques involved in its analysis.