Long-time asymptotics of solutions to the Keller–Rubinow model for Liesegang rings in the fast reaction limit

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We consider the Keller-Rubinow model for Liesegang precipitation patterns in one spatial dimension in the fast reaction limit as introduced by Hilhorst, van der Hout, Mimura, and Ohnishi (J. Stat. Phys., 2009). We conjecture that solutions to this model converge, independent of the initial concentration, to a universal profile for large times in parabolic similarity coordinates. The candidate limit profile is necessarily thes the solution of a certain one-dimensional boundary value problem which can be solved explicitly. Depending on the strength of the source, there are two nontrivial regimes. In the first, the *transitional regime*, precipitation is restricted to a bounded region in space and the concentration converges to a single unique profile. In the second, the *supercritical regime*, the concentration converges to one of a one-parameter family of asymptotic profiles, selected by a solvability condition for the one-dimensional boundary value problem. Here, our convergence result is only conditional: we prove that if convergence happens, either pointwise for the concentration or in an averaged sense for the precipitation function, then the other field converges likewise; convergence in concentration is uniform, and the asymptotic profile is indeed the profile selected by the solvability condition. We demonstrate numerically that the solution behaves indeed as suggested by the theorem.