

# The Schläfli Fan

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In 1849, Arthur Cayley and George Salmon proved one of the most famous results in algebraic geometry: every smooth cubic surface contains exactly 27 lines. In tropical geometry algebraic surfaces are replaced with polyhedral complexes of dimension two. Since early development of this recent mathematical field, two natural problems were to understand whether the aforementioned statement holds for smooth tropical cubic surfaces and to classify combinatorial positions of their tropical lines. The answer to the first turned out to be false, as tropical surfaces might contain families of tropical lines. Moreover, classifying positions of tropical lines reveals computational challenges due to the massive number of combinatorial types of smooth tropical cubic surfaces. The latter are parametrized by 14 373 645 symmetry classes of maximal cones in the unimodular secondary fan of the triple tetrahedron.

In this talk, after introducing tropical surfaces and their lines, we will look at the Schläfli fan which gives a further refinement of these cones. It reveals all possible patterns of the 27 or more lines on tropical cubic surfaces, thus serving as a combinatorial base space for the universal Fano variety.