Large-time behaviour of solutions of parabolic equations on the real line with convergent initial data

Antoine Pauthier[1,*], **Peter Poláčik**[1]
[1] *School of Mathematics, University of Minnesota*
*Email: apauthie@umn.edu

We consider the following semilinear parabolic equation

$$\partial_t u = \partial_{xx} u + f(u) \tag{1}$$

where f is a Lipschitz function, with a bounded initial condition. If the solution $u(\cdot, t)$ is bounded, then it is a classical solution, defined for all $t \geq 0$, and it is well known that the set of all limit profiles $\omega(u) := \{\varphi \in L^\infty(\mathbb{R}),\ u(t_n, \cdot) \to \varphi \text{ in } L^\infty_{loc}(\mathbb{R}),\ \text{for some sequence } t_n \to \infty\}$ is non-empty and connected. One can wonder to what extend these limit profiles, and therefore the long-time behaviour of solutions, is determined by the stationary equation. If the solution is convergent, for instance, the $\omega-$limit set is reduced to a single element, stationary solution of the equation. Convergence is of course not the general behaviour for such an equation, but we can expect that all the limit profiles $\varphi \in \omega(u)$ are steady states for the initial equation. The solution is then said to be *quasiconvergent.*

In this talk, I will present some general quasiconvergence results when the initial condition admits finite limits at $x = \pm\infty$. In particular, in the generic situation $u_0(-\infty) \neq u_0(+\infty)$, any bounded solution is quasiconvergent, independently of the nonlinear term f. In a second part, we focus on the situation where the limits are equal, $u_0(-\infty) = u_0(+\infty)$. A similar result is impossible, due to known counter-examples. Assuming further non-degeneracy assumtions, we prove that these counter-examples are the only situations where quasiconvergence may fail to happen.