A variant of Wall's bordism exact sequence for spin manifolds

Julian Poedtke^{1,*}

¹Department of Mathematics, Muenster, Germany *Email: j_poed01@uni-muenster.de

This master thesis relates Spin- and $Spin^c$ -bordism in a similar way as M. F. Atiyah ([1]) and C. T. C. Wall ([2]) related oriented and non-oriented bordism. The heart piece builds the following long exact sequence relating Spin- and $Spin^c$ -bordism by a geometrically constructed operator ∂^1 :

$$\ldots \longrightarrow \Omega_k^{Spin} \longrightarrow W_k \xrightarrow{\partial^1} \Omega_{k-2}^{Spin} \longrightarrow \Omega_{k-1}^{Spin} \longrightarrow \ldots$$

Where W_k is a subgroup of $\Omega_k^{Spin^c}$. Various other sequences get deduced and a structural result about the $Spin^c$ -bordism theory gets proven. Namely that there is a geometric splitting for $k \ge 4$

$$\Omega_k^{Spin^c} \cong W_k \oplus \Omega_{k-4}^{Spin^c}$$

To derive these results the $Spin^c$ -bordism with coefficients theory gets introduced and a duality theorem gets found by a Pontrjagin-Thom construction. Lastly, the developed theory and results get interpreted homotopy theoretically.

References

- [1] M. F. Atiyah, Bordism and cobordism, Proc. Cambridge Philos. Soc. 57 (1961), pp. 200–208.
- [2] C. T. C. Wall, Determination of the cobordism ring, Annals of Mathematics. Second Series 72 (1960), pp. 292–311.