The problem of existence of static and electrostatic solutions of the Einstein equations in arbitrary topology

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Vacuum static black hole solutions of the Einstein equations are central in General Relativity and important in geometry. A recent classification showed that S^1 -symmetric black holes can be only of three kinds: Schwarzschild, Boost, or Myers/Korotkin-Nicolai, each family having its distinct topology type. We will explain that, while Schwarzschild's topology admits charged static S^1 -symmetric black holes, those of the Boosts and Myers/Korotkin-Nicolai do not. Black holes in such topologies cannot hold a charge. The proof of this peculiar fact is done by first transforming the static solution into a vacuum stationary solution by means of a Kramer-Neugebauer transform, then proving decay estimates using techniques a la Bakry-Émery, and finally showing that too much electrostatic energy would concentrate at infinity, thus reaching an impossibility. The type of result, as well as the techniques, appear to be new.