Spaces and Moduli Spaces of Riemannian Metrics with Positive Scalar Curvature

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A fundamental problem in Riemannian geometry is the question, whether a given manifold can be curved in a specific way, i.e. whether it admits a Riemannian metric satisfying certain curvature conditions. If yes, one can further ask how many there are and how they are related to each other. This leads to the following construction: For a given manifold one considers the space of all Riemannian metrics satisfying the required conditions. The moduli space is then obtained by identifying metrics which are isometric. To understand the topology of these spaces is a very hard problem in general.

We will consider spaces and moduli spaces of Riemannian metrics with positive scalar curvature. Methods from surgery theory and the index theory of Dirac operators have led to various results on the number of path components of these spaces. The goal of this talk is to explain this approach. For that we will focus on a result given in [1], which states that both spaces have an infinite number of path components for certain manifolds with finite fundamental group whose dimension is odd and at least five. We will furthermore consider applications and extensions of this theorem.

References

[1] B. Botvinnik and P. B. Gilkey, The eta invariant and metrics of positive scalar curvature *Mathematische Annalen* 302.3 (1995), pp. 507–517.