

Unexpected behaviour of fundamental solutions of general higher-order elliptic operators

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The positivity preserving property for second-order elliptic equations ($Lu = f \geq 0 \Rightarrow u \geq 0$?) is a well-known consequence of the maximum principle. In the higher-order setting such expected behaviour is often spoiled by the influence of the boundary conditions, and in general the answer is negative. However, one still expects that when applying an extremely concentrated right-hand side – a δ -distribution – then close to this point the solution should respond in the same direction. Such *local question* can be rewritten in terms of positivity of a suitable singular fundamental solution near to its pole, and it is known for the polyharmonic operator $(-\Delta)^m$, or - more in general - for powers of second-order operators, see [1]. In the work [2], we show that such results cannot be in general extended for any elliptic higher-order operator. Indeed, by means of explicit formulae for the singular fundamental solutions, we prove that positivity near the unbounded singularity persists only in special dimensions, while in general the behaviour is sign-changing.

References

- [1] H.-Ch. Grunau, F. Robert, Positivity and almost positivity of biharmonic Green's functions under Dirichlet boundary conditions, *Arch. Rational Mech. Anal.* **195** (2010), 865–898.
- [2] H.-Ch. Grunau, G. Romani, Unexpected differences between fundamental solutions of general higher-order elliptic operators and of products of second-order operators, arXiv:1902.06503.