What does it mean for an infinite graph to be Hamiltonian?

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The study of *Hamiltonian graphs*, i.e. finite graphs having a cycle that contains all vertices of the graph, is a central theme of finite graph theory. For infinite graphs such a definition cannot work, since cycles are finite. We shall debate possible concepts of Hamiltonicity for infinite graphs and eventually follow the topological approach by Diestel and Kühn [1,2], which allows to generalise several results about being a Hamiltonian graph to locally finite graphs, i.e. graphs where each vertex has finite degree.

An *infinite cycle* of a locally finite connected graph G is defined as a homeomorphic image of the unit circle $S^1 \subseteq \mathbb{R}^2$ in the Freudenthal compactification $|G|$ of G. Now we call G *Hamiltonian* if there is an infinite cycle in $|G|$ containing all vertices of G.

We shall examine how we can force graphs to be Hamiltonian via forbidden induced subgraphs. We extended to locally finite graphs several sufficient conditions for finite graphs to be Hamiltonian. Our results are about claw- and net-free graphs, claw- and bull-free graphs, but also about further graph classes being structurally richer. In this talk we introduce, debate and motivate the topological viewpoint and sketch the proofs for the results mentioned above.

References

[1] R. Diestel and D. Kühn, On infinite cycles I, *Combinatorica* **24** (2004), pp. 69–89.

[2] R. Diestel and D. Kühn, On infinite cycles II, *Combinatorica* **24** (2004), pp. 91–116.