Bifurcations of a cubic Helmholtz system

Dominic Scheider[1,*], **Rainer Mandel**[1]

[1] *Karlsruhe Institute of Technology, Karlsruhe, Germany*

[*] Email: `dominic.scheider@kit.edu`

In this talk I will present an existence result for localized vector solutions of the cubic Helmholtz system

$$\begin{aligned} -\Delta u - \mu u &= u^3 + buv^2 \qquad \text{in } \mathbb{R}^3, \\ -\Delta v - \nu u &= v^3 + bvu^2 \qquad \text{in } \mathbb{R}^3 \end{aligned}$$

for given $\mu, \nu > 0$ and a coupling parameter $b \in \mathbb{R}$. It is obtained using bifurcation from a simple eigenvalue and by analyzing the asymptotic behavior of the solutions in the far field, i.e. the leading order of the asymptotic expansion of $u(x), v(x)$ as $|x| \to \infty$.

I will then show how these methods can be applied to construct solutions of the cubic Klein-Gordon equation

$$\partial_t^2 U(t,x) - \Delta U(t,x) + m^2 U(t,x) = U(t,x)^3, \qquad (t,x) \in \mathbb{R} \times \mathbb{R}^3.$$

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References

[1] R. Mandel, D. Scheider: Bifurcations of nontrivial solutions of a cubic Helmholtz system, *Preprint*, `https://arxiv.org/abs/1710.06332`, accepted for publication in ANONA.