Highly linked tournaments with large minimum out-degree

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Given a positive integer k, a directed graph is said to be *k-linked* if for any two disjoint sets of vertices $\{x_1, \ldots, x_k\}$ and $\{y_1, \ldots, y_k\}$ there are vertex disjoint directed paths $P_1, \ldots, P_k$ such that P_i joins x_i to y_i for $i = 1, \ldots, k$. Clearly, k-linkedness is a stronger notion than the usual notion of strong k-connectivity. But how much stronger is it? Thomassen constructed directed graphs with arbitrarily large connectivity that are not even 2-linked. It is natural, therefore, to address this question in the restricted setting of tournaments. Resolving a conjecture of Kühn, Lapinskas, Osthus, and Patel, Pokrovskiy showed that any $452k$-strongly-connected tournament is k-linked. He further conjectured, in analogy with the situation for undirected graphs, that there is a function $f : \mathbb{N} \to \mathbb{N}$ such that any $2k$-strongly-connected tournament with minimum in and out-degree at least $f(k)$ is k-linked. In this talk, I shall present some recent progress made on this conjecture.