Binary quadratic forms and Apollonian circle packings

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If f is a positive definite binary quadratic form with integer coefficients, how many numbers smaller than some positive X does f represent and with what multiplicity? For forms with fundamental discriminants, V. Blomer and A. Granville [1] answered this by giving asymptotics uniform in the discriminant. These results have an important application in a question regarding Apollonian circle packings. J. Bourgain and E. Fuchs [2] make use of the uniformity in the discriminant to prove that the integers appearing as curvatures in an arbitrary integer Apollonian circle packing form a set of positive density inside the natural numbers.

As it was given, the proof in [2] requires a generalisation of the asymptotics in [1], since we need to consider quadratic forms with non-fundamental discriminants and restrict to counting proper representations. My talk will sketch the way we can obtain the necessary results after giving a general introduction to these topics.

References

- V. Blomer and A. Granville, Estimates for representation numbers of quadratic forms, *Duke Mathematical Journal* 135 (2006), pp. 261 302
- [2] J. Bourgain and E. Fuchs, A proof of the positive density conjecture for integer Apollonian circle packings, J. Amer. Math. Soc. 24 (2011), pp. 945–967