## The Hard Lefschetz Theorem, history and recent progress

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The Hard Lefschetz Theorem is a corner stone of complex analytic geometry and arithmetic geometry. Lefschetz' own original proof was not complete. Hodge gave the first proof using harmonic theory. Deligne developed the theory of weights to give a proof which then holds in positive characteristic. On the complex side, harmonic theory enabled Simpson to generalize Hard Lefschetz to semi-simple complex local systems. We'll report on recent progress on the arithmetic side. (The new material is joint work with Moritz Kerz).