Symplectic geometry of Weinstein domains

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Contact geometry is often considered to be the odd dimensional counterpart to symplectic geometry, since contact manifolds sometimes arise naturally as boundaries of symplectic manifolds. What structures on a symplectic manifold do we need in order to determine if the boundary carries a natural contact structure? To give an example we need the following definition - a Liouville vector field is a vector field X_L on a symplectic manifold (W, ω) satisfying $\omega = \mathcal{L}_{X_L}\omega$. It can be shown that if (W, ω) is a compact manifold with boundary and X_L a Liouville vector field on W pointing transversely outward at the boundary, ∂W inherits a contact structure.

What about the converse? If (M, ξ) is a contact manifold, does there exist a symplectic filling for M, that is a compact symplectic manifold (W, ω), such that $M = \partial W$ and the contact structure on M is compatible with the symplectic structure on W? The answer is not straightforward. To begin with, there are different types of symplectic fillings depending on what it means for the contact structure to be compatible with the symplectic structure - weak, strong and Weinstein fillings. In my talk I will introduce the three diffretent types of symplectic fillings and sketch the proof of the following theorem:

Theorem: If M is an aspherical contact manifold of dimension $n \geq 5$, then it does not admit a Weinstein filling.

This result will be used to compare weak and Weinstein fillable contact manifolds.