Gradient systems and evolutionary Γ-convergence

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Many ordinary and partial differential equations can be written as a gradient flow, which means that there is an energy functional that drives the evolution of the the solutions by flowing down in the energy landscape. The gradient is given in terms of a dissipation structure, which in the simplest case is a Riemannian metric. We will highlight classical and nontrivial new examples in reaction-diffusion systems or dissipative quantum mechanics. We will emphasize that providing a gradient structure for a given differential equation means that we add additional physical information.

Considering a family of gradient systems depending on a small parameter, it is natural to ask for the limiting or effective gradient system describing the evolution in the limit of the parameter tending to 0. This can be achieved on the basis of De Giorgi's Energy-Dissipation Principle. We discuss the new notion of "EDP convergence" and show by examples that this theory is flexible enough to allow for situations where starting from a linear Riemannian structure (or quadratic dissipation potentials) we arrive at physically relevant, nonlinear effective kinetic relations, namely exponential laws for transmission at membranes or slip-stick motion on rough surfaces.