On the structure of graphs without forbidden induced subgraphs

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The Erdős-Hajnal conjecture is one of the most challenging open conjectures in graph theory, which is a sub-branch of discrete mathematics. The conjecture asks if for every graph H there exists a constant ϵ, such that every graph on n vertices that does not contain H as an induced subgraph contains either a clique or an independent set of size n^{ϵ}. If such an ϵ exists for a graph H, one can also ask for the 'best' such exponent and define $\epsilon(H)$ as the supremum over all $\epsilon > 0$ for which the conjecture holds.

A bipartite version of the conjecture asks if for every bipartite graph H there exists a constant $\widetilde{\epsilon}(H)$, such that every bipartite graph G with n vertices in each part which does not contain H as an induced subgraph has a complete/empty bipartite subgraph, with parts of size $n^{\widetilde{\epsilon}(H)}$. This version of the conjecture is known to be true, but one can again ask about the 'best' $\widetilde{\epsilon}(H)$ for given bipartite H, which has not been addressed before. We determine values for $\widetilde{\epsilon}(H)$ for some small graphs H. We also answer the question for which forbidden graphs H $\widetilde{\epsilon}(H)$ is linear in n, except for 4 graphs.

References

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[2] P. Erdős and A. Hajnal, Ramsey-type theorems, *Discrete Applied Mathematics* **25:1-2** (1989), pp. 37–52.